

# SPLIT UED

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Kong, SCP, Rizzo, JHEP 1004:081, 2010  
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# IPMU $\neq$ IMPU



Piazza Fujiwara @ IPMU

## Quiz)

- KK-parity requires flat geometry.  
Yes or No?
- KK-parity forbids 5D fermion mass.  
Yes or No?

# Contents

- UED KK-parity
- Split-UED KK-spectrum with bulk masses for fermions
- Split-UED' UED as an effective theory of RS model

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- UED KK-parity
- Split-UED KK-spectrum with bulk masses for fermions
- Split-UED' UED as an effective theory of RS model

2001-2009-2010

More ambitious

# UED

Appelquist, Cheng, Dobrescu 2001

- UED models are models with flat, compact extra dimension,  $S^1/Z_2$ , in which all fields propagate.
- The SM is identified with the theory of lowest-lying modes.
- A discrete  $Z_2$  symmetry, called **KK-parity**, is conserved even though KK-number conservation is broken by fixed points. (more comes later)

# UED

- Allowing only flat profiles in UED,  $L/R > O(300)$  GeV is not excluded by EWPT.
- UED is MFV (assuming no brane localized flavor asymmetric operators). Flavor structure is given by Yukawa.
- A minimal version of UED (MUED) has few new parameters ( $R, \Lambda$ ) thus easy to scan the parameter space. Brane localized operators are assumed to vanishes at cutoff scale  $\Lambda$ .
- With **KK-parity**, UED mimics MSSM+R-parity. Some call UED “Bosonic SUSY”. An interesting bench mark model from the perspective of model discrimination at the LHC.



# ● ..but what's the use of it?

- ✧ UED does not solve any problem in the SM except providing a new DM candidate, again, thanks to **KK-parity**.
- ✧ We want to extend UED and make it is useful and interesting. Definitely **we want to keep KK-parity**.



Before considering  
extension of MUED,  
let's consider

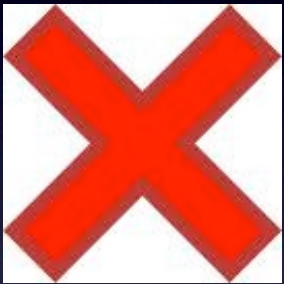
# KK-parity



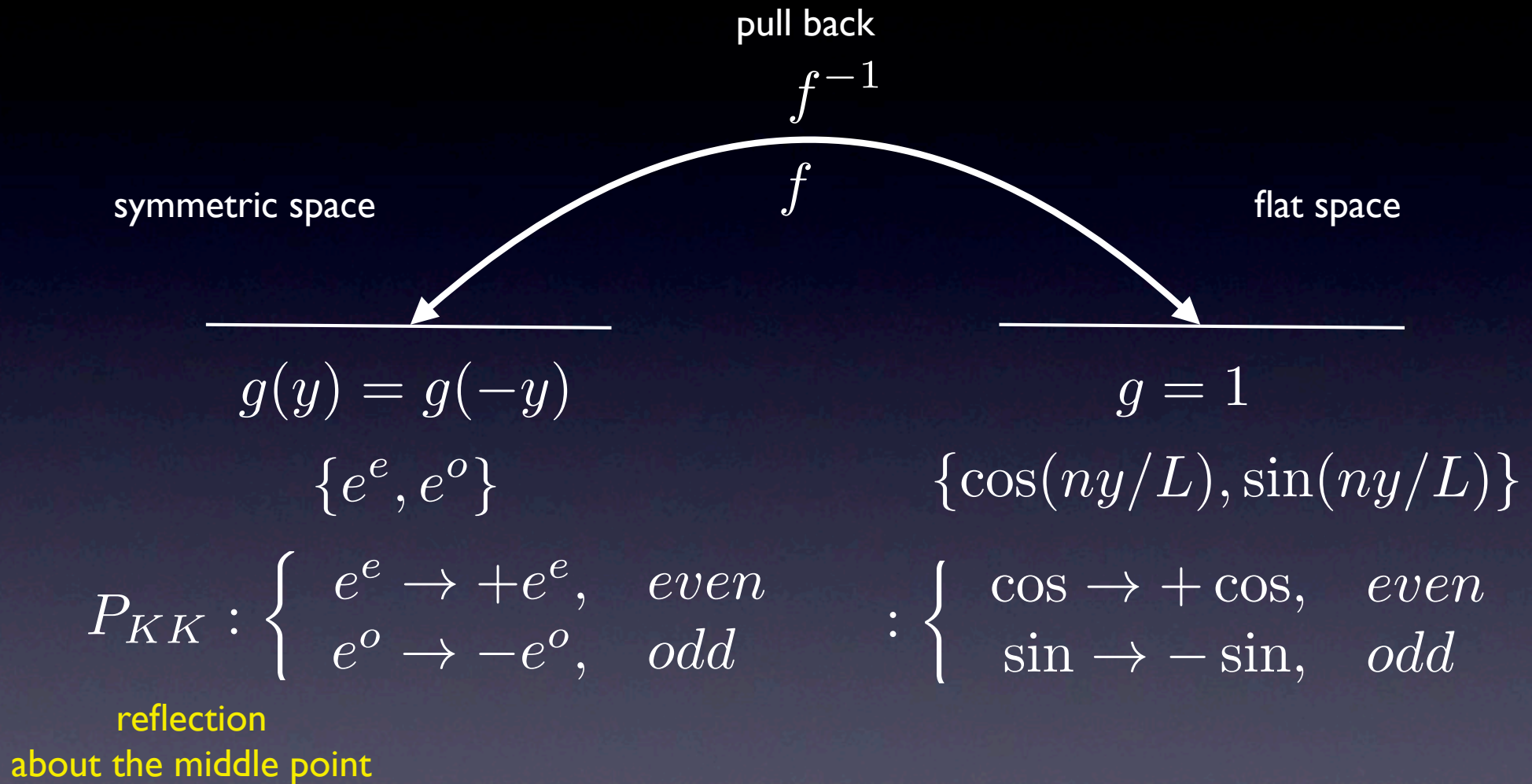
- $Z_2$  reflection about the middle point of extra dimension.
- A remnant symmetry of 5D translational invariance, which is broken by end points (or fixed points in orbifold language).
- It is often claimed that KK-parity requires flat geometry like in UED. Yes or No?

- KK-parity requires flat geometry.  
Yes or No?

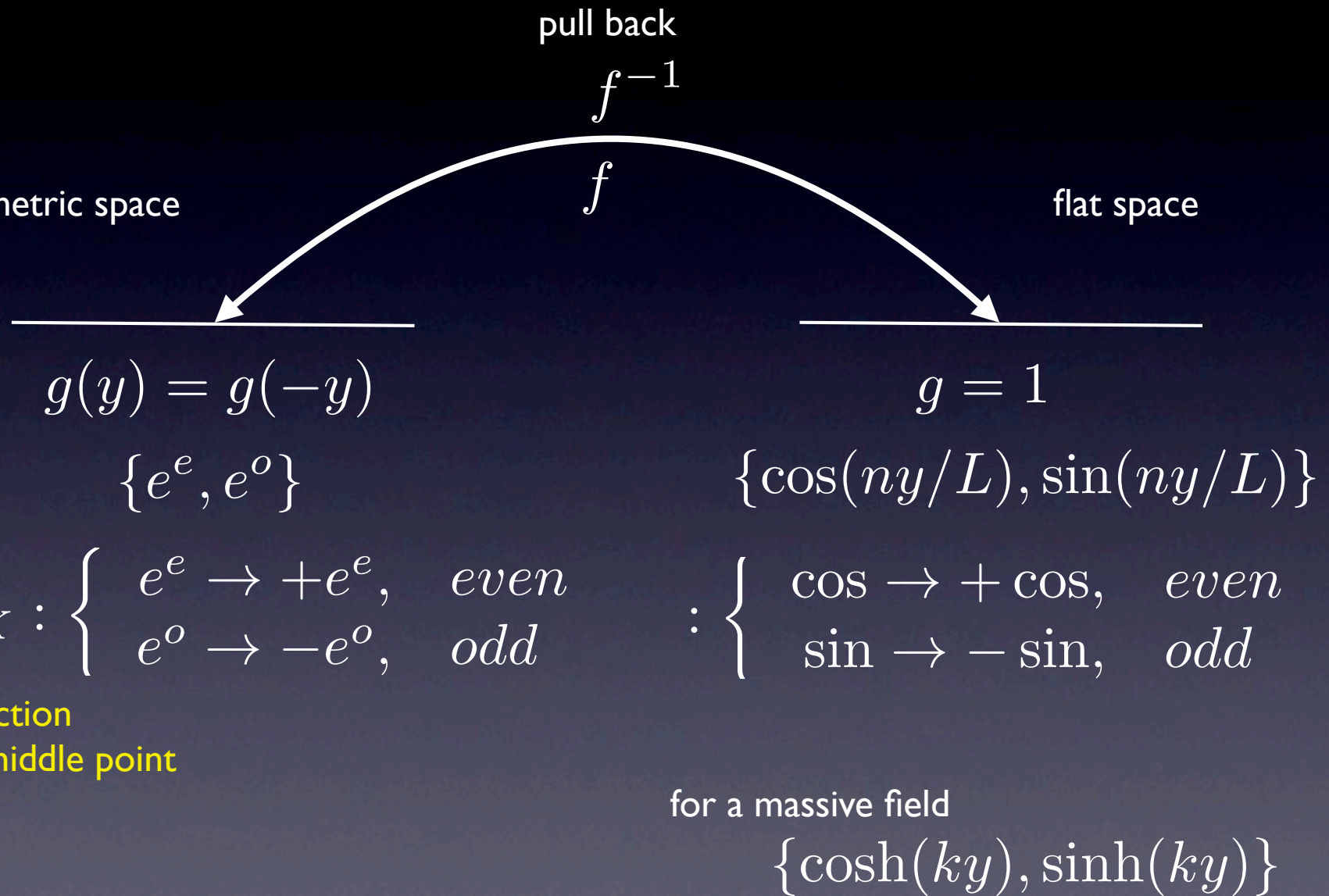
- KK-parity requires flat geometry.  
Yes or No?



Indeed, KK-parity can be defined  
on any symmetric space.



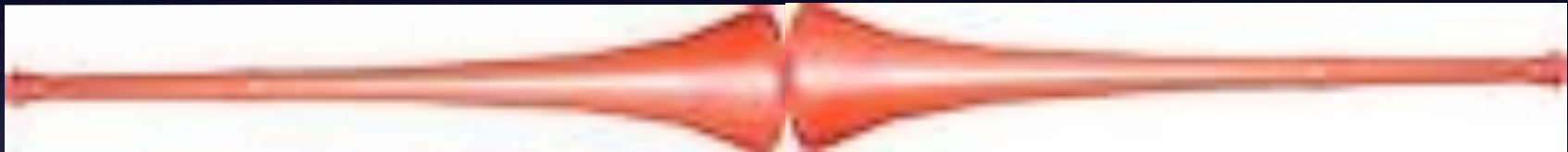
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# example spaces

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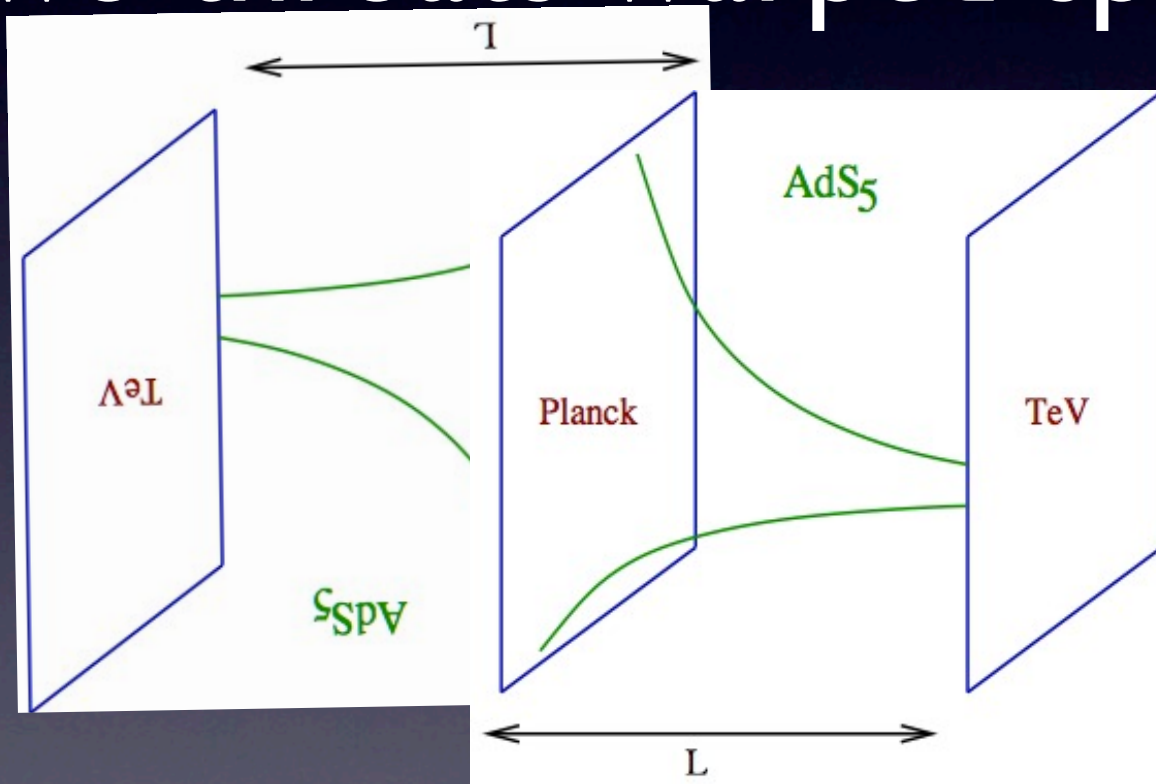
## Bubuzela space



Two Bubuzelas glued together, FIFA2010

# example spaces

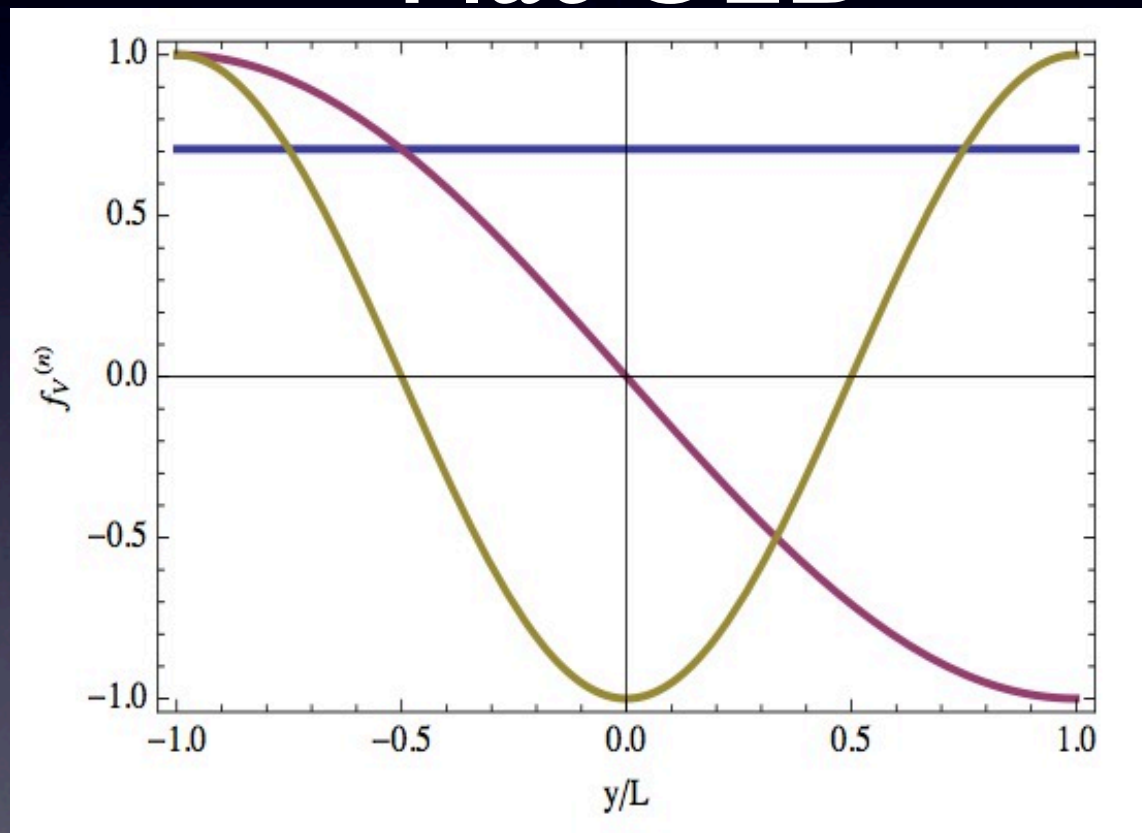
## Two throats warped space



Agashe, Falkowski, Low, Servant (2008)

# example spaces

## Flat UED

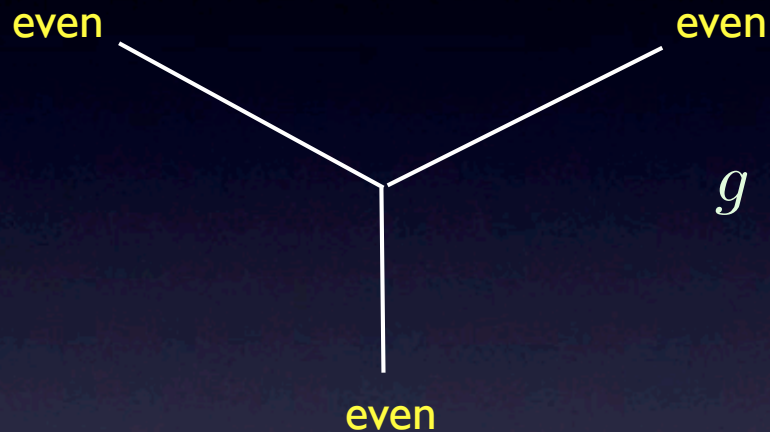


$n=0, 2$ : even

$n=1$ : odd

$$P_{kk}=(-1)^n$$

# Interaction allowed/forbidden



$$g \propto \int_{-L}^L dy \psi_{\text{even}} \psi_{\text{even}} \psi_{\text{even}} \neq 0$$

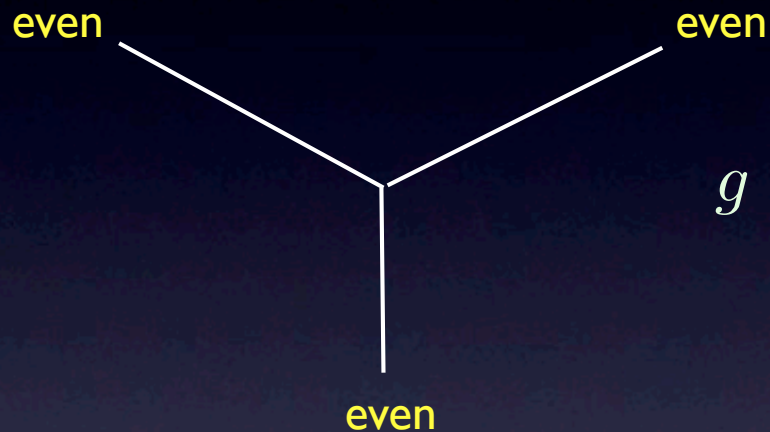
Allowed



$$g \propto \int_{-L}^L dy \psi_{\text{odd}} \psi_{\text{even}} \psi_{\text{even}} = 0$$

Forbidden

# Interaction allowed/forbidden



$$g \propto \int_{-L}^L dy \psi_{\text{even}} \psi_{\text{even}} \psi_{\text{even}} \neq 0$$

Allowed

odd even

Underlying Math:

An odd function cannot be decomposed  
into finite number of even functions

= 0

even

# Minimal Extensions

Without introducing additional field contents to UED, we can extend the model by introducing following terms:

- Brane localized terms (Dim=5, 6) Carena, Tait, Wagner (2002)
- Bulk mass for fermion (Dim=4) SCP, Shu (2009) “split UED”

# Bulk Mass

- $M_{\text{gauge}}=0$  : gauge symmetry.  $m_5 \bar{\psi}_L \psi_R + (L \rightarrow R)$
- $M_{\text{fermi}} \neq 0$  : Vectorlike mass term for fermion is not forbidden by 5D Lorentz symmetry and gauge symmetry. In principle, this term should be included in effective theory point of view!
- It has been often claimed that KK-parity forbids the bulk mass term. But, is it really right?

- KK-parity forbids 5D fermion mass. Yes or No?

- KK-parity forbids 5D fermion mass. **Yes or No?**



and



**Dirac Bilinear is odd** under the reflection thus KK parity forbids KK-even mass **allows KK-odd mass.**

$$y \rightarrow -y$$

$$\Psi(x^\mu, y) \rightarrow \pm \gamma_5 \Psi(x^\mu, y)$$

$$m_5(y) \rightarrow m_5(-y) = -m_5(y)$$

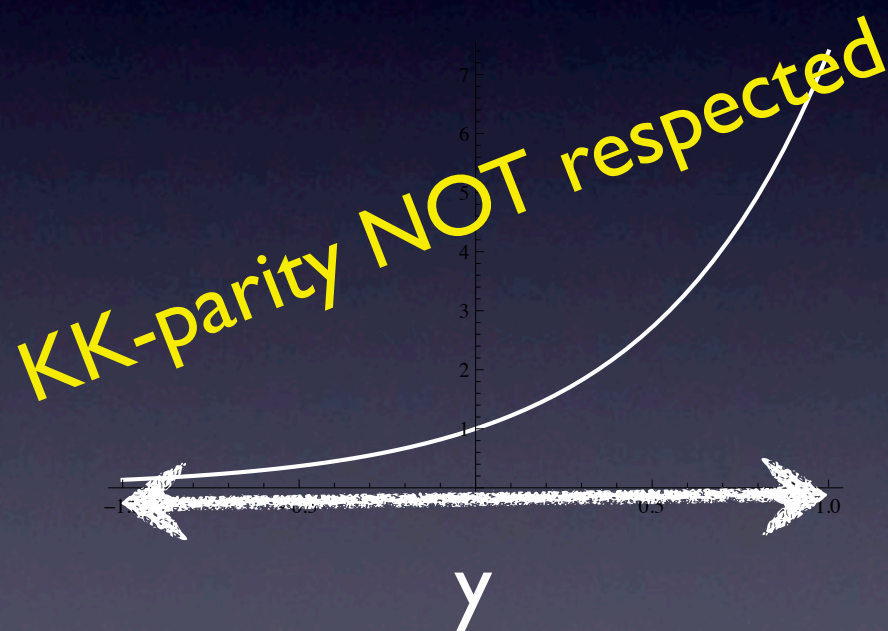
$$m_5 \bar{\Psi} \Psi \text{ is invariant}$$

(proof)

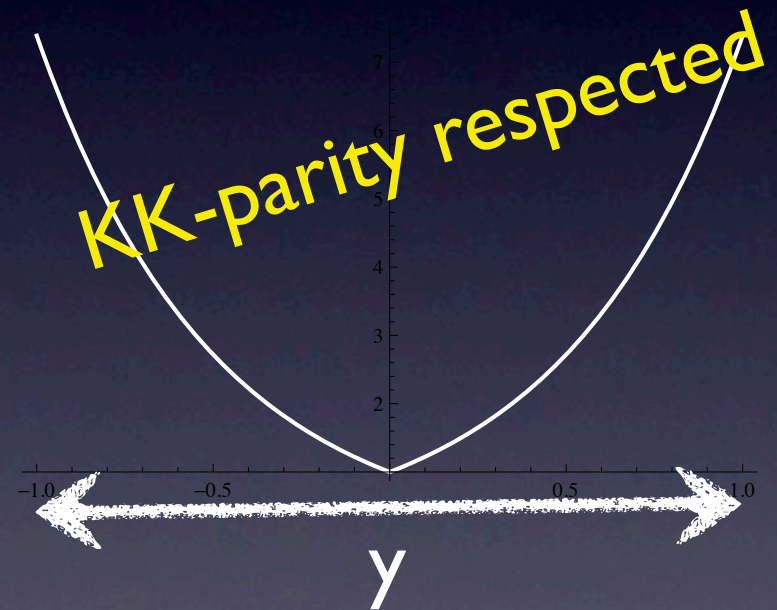
$$\begin{aligned} \bar{\Psi} \Psi &\rightarrow (\gamma_5 \Psi)^\dagger \gamma^0 (\gamma_5 \Psi) \\ &= \Psi^\dagger \gamma_5 \gamma^0 \gamma_5 \Psi \\ &= -\Psi^\dagger \gamma^0 \Psi \\ &= -\bar{\Psi} \Psi \end{aligned}$$

$$(\partial_y \pm m) f_{R/L}^{(0)} = 0$$

with “even mass”  
 $m(-y)=m(y)$

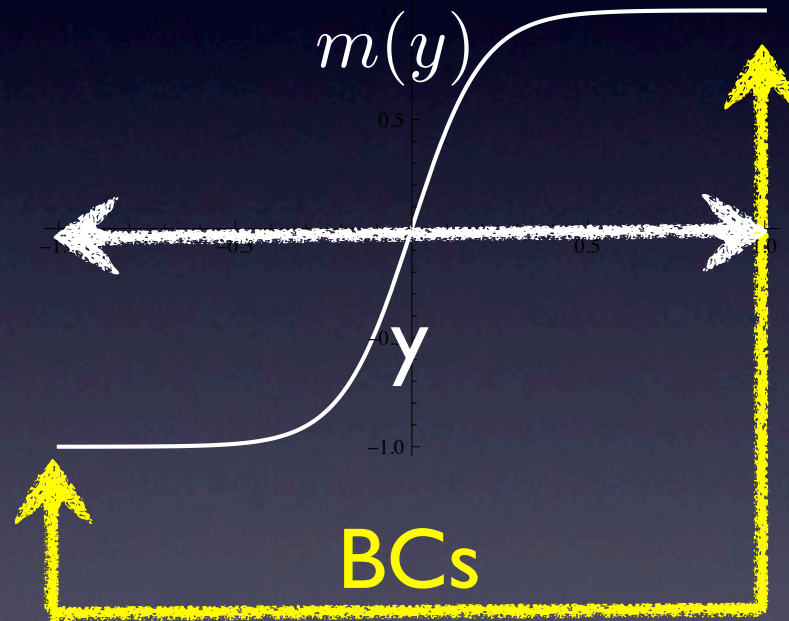


with “odd mass”  
 $m(-y)=-m(y)$



# Odd mass on orbifold

$$M_5(y) \rightarrow M_5(-y) = -M_5(y)$$



The lowest energy configuration  
interpolating boundary values:  $+M, -M$

$$M \tanh \mu y \rightarrow M \theta(y)$$

Georgi, Grant, Hailu (2001)

# Split UED

SCP, Shu, 2009

$$\Delta S = - \int d^5 x \mu \theta(y) \bar{\psi} \psi$$

- With the odd bulk mass, chiral zero mode remains massless but the profile of zero mode is exponentially localized.

$$f_{R/L}^{(0)} = \sqrt{\frac{\pm \mu}{1 - e^{\mp 2\mu L}}} e^{\mp \mu y}$$

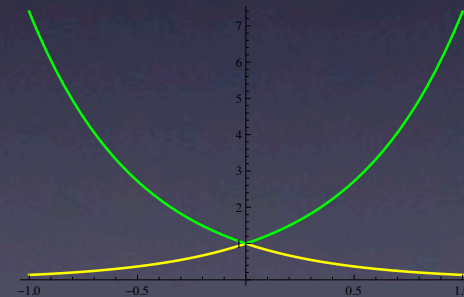
- KK-masses are deformed

$$m_n = \sqrt{\mu^2 + k_n^2}$$

$$n = 2, 4, \dots : k_n = \frac{n\pi}{L}$$

$$n = (1, )3, \dots : k_n = \mp \mu \tan k_n L$$

for DL/DR or RH/LH zero mode



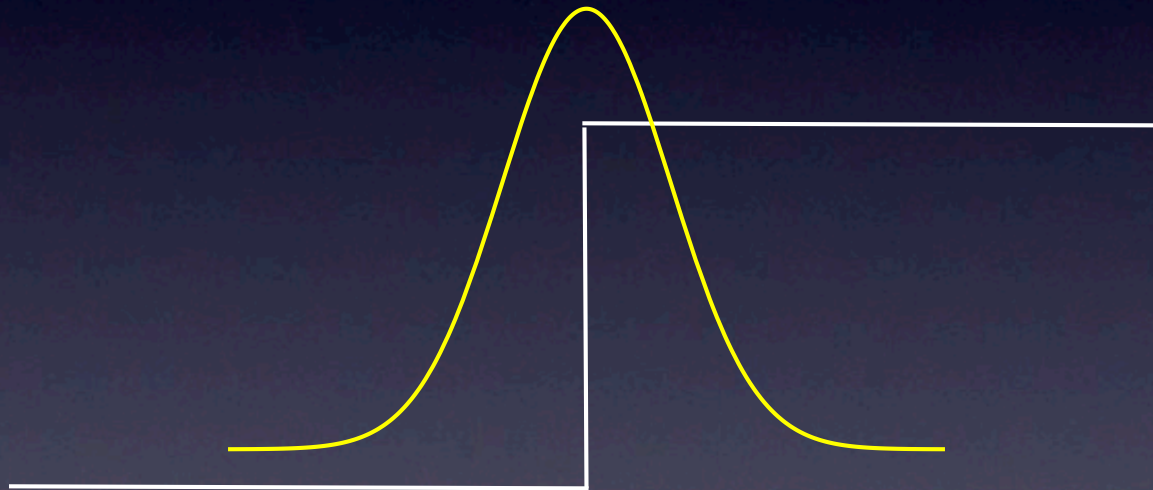
- There can exist a new ultralight mode.

reminder

# Domain wall fermion

$$[-\infty, +\infty]$$

- A 'trapped fermion' exists in the presence of domain wall in infinite extra dimension. It is chiral (=massless). [domain wall fermion]



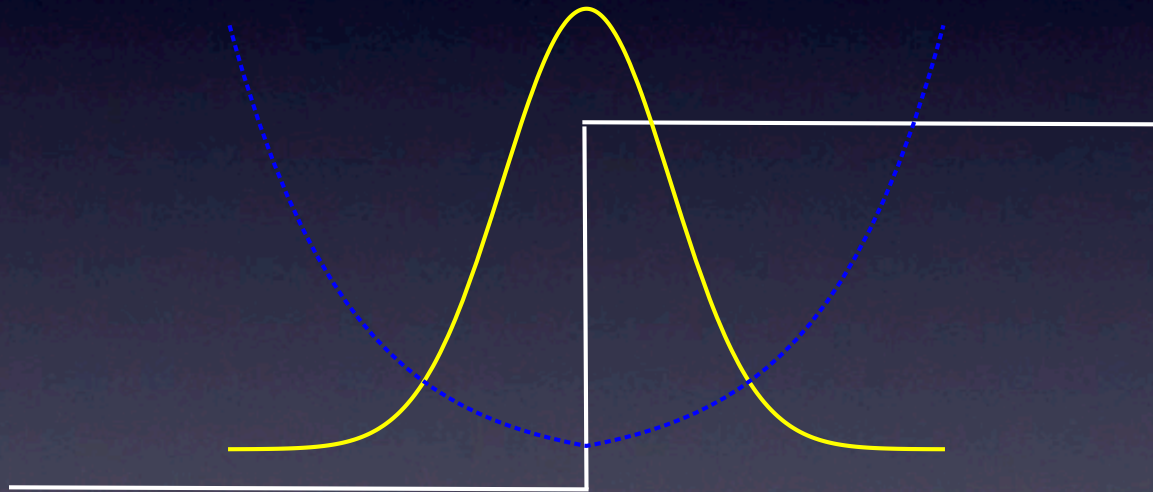
- The other chiral state is exponentially diverging (non-normalizable mode), which is not physical mode.

Only domain wall fermion is physical.

# Domain wall fermion

$$[-L, +L]$$

- A 'trapped fermion' still exists in the presence of domain wall in finite extra dimension. It is chiral (=massless). [domain wall fermion]



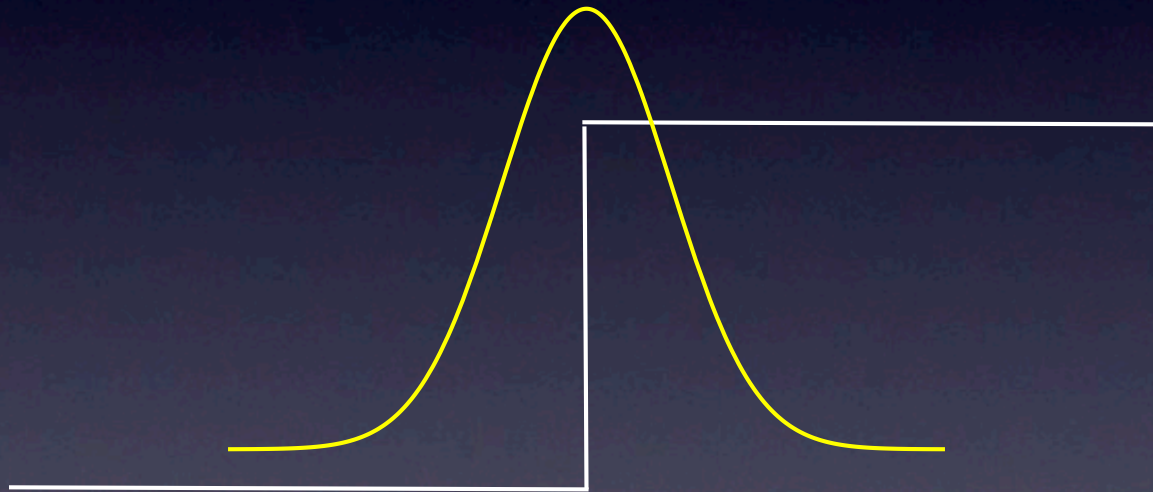
- The other chiral state is exponentially growing but normalizable since the extra dimension is finite. This mode is also physical.

Both [can be] physical

# Domain wall fermion

$$[-L, +L]$$

- There are two choices of BCs on orbifold.  
(i) Dirichlet BC for growing mode. $\Rightarrow$  Domain wall fermion is physical zero mode

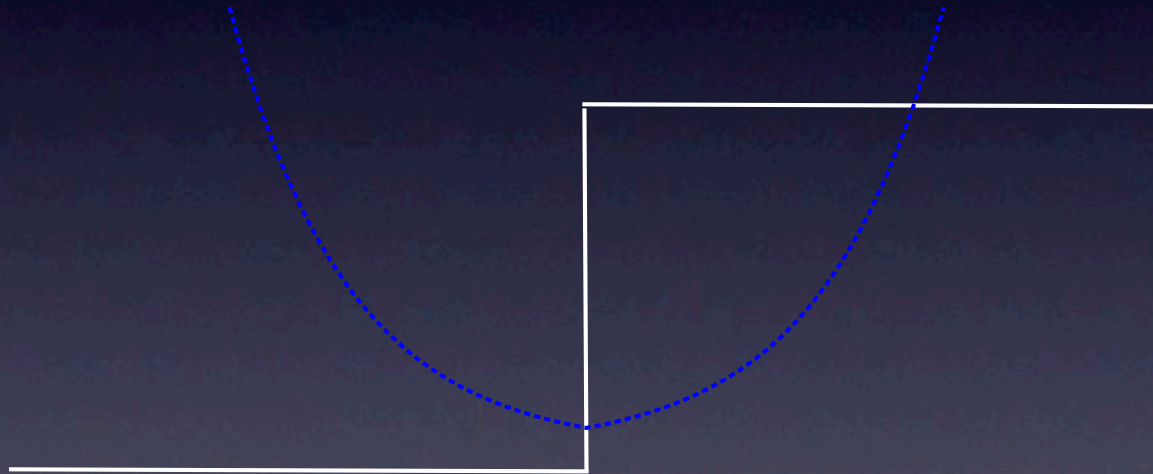


This case is totally OK as the domain wall fermion is a natural chiral zero mode

# Domain wall fermion

$$[-L, +L]$$

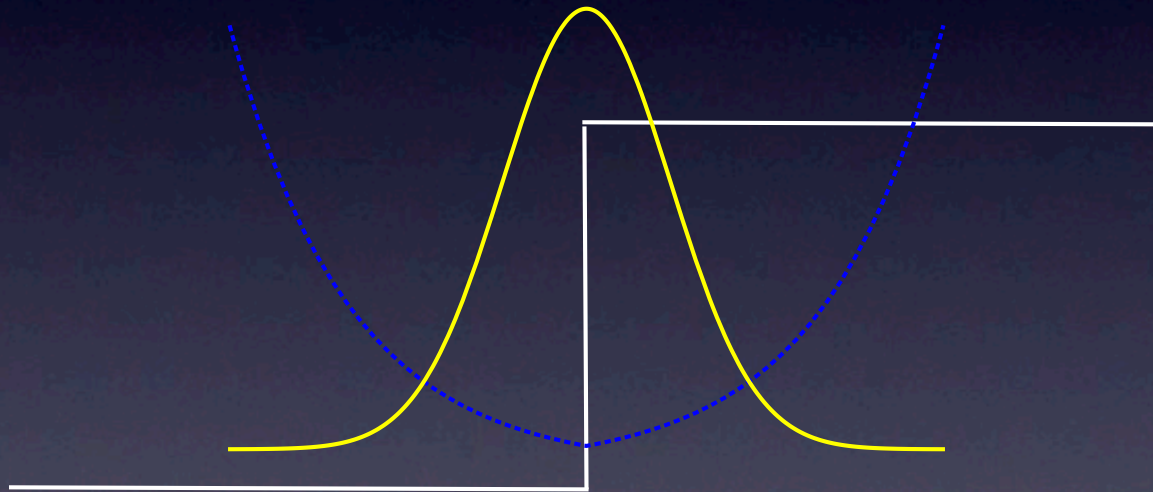
- There are two choices of BCs orbifold.  
(ii) Dirichlet BC for Domain wall mode. $\Rightarrow$  Growing mode is physical zero mode



# Domain wall fermion

$$[-L, +L]$$

- There are two choices of BCs orbifold.
- (ii) Dirichlet BC for Domain wall mode. $\Rightarrow$  Growing mode is physical zero mode

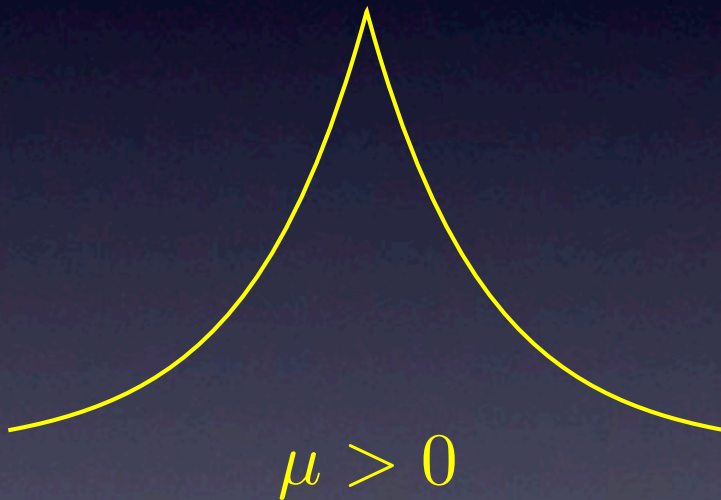


In this case, actually, 1<sup>st</sup> excited KK mode will become the 'would-be' domain wall fermion', which is very light.  $m_1 = 2|\mu|e^{-\mu L}$

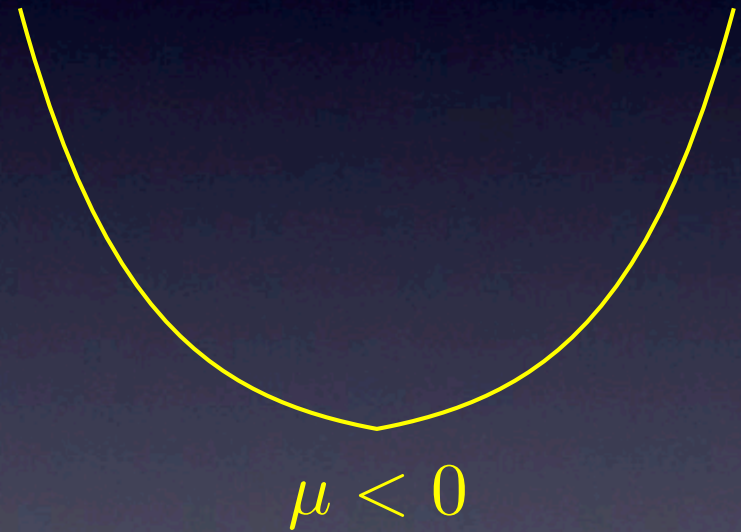
# DL

When Dirichlet BC is imposed to L-handed chirality (DL), R-handed chiral zero mode is the solution:

$$f_R^{(0)} = \sqrt{\frac{\mu}{1 - e^{-2\mu L}}} e^{-\mu|y|}$$



no ultralight mode

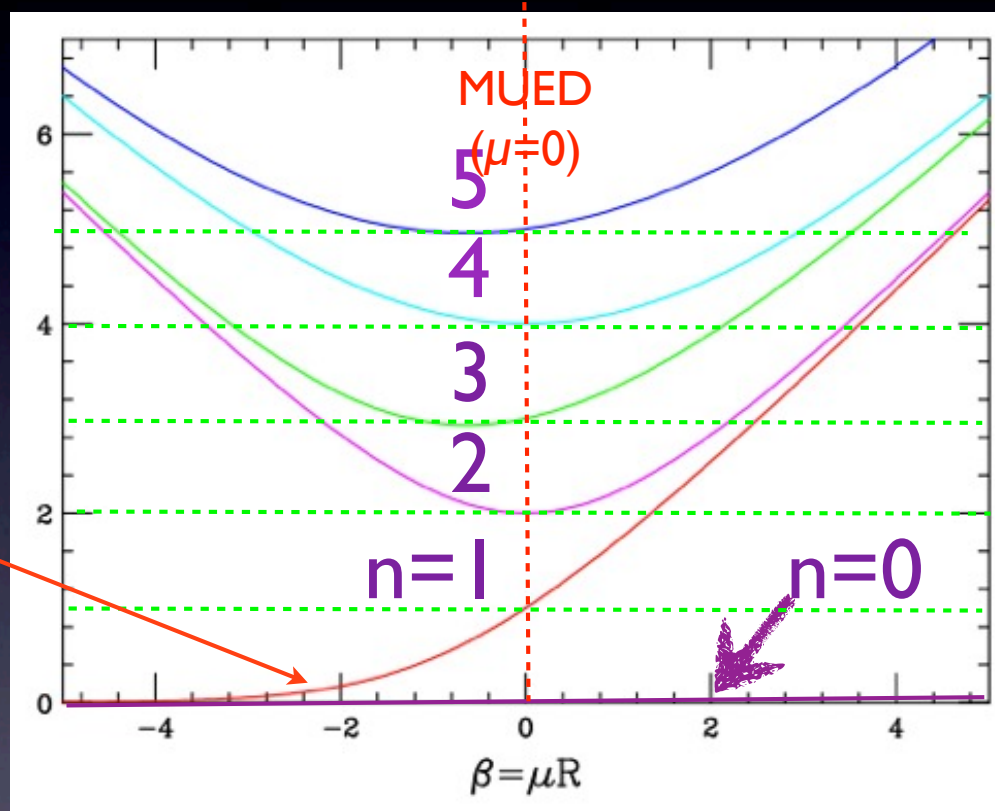


+ultralight mode

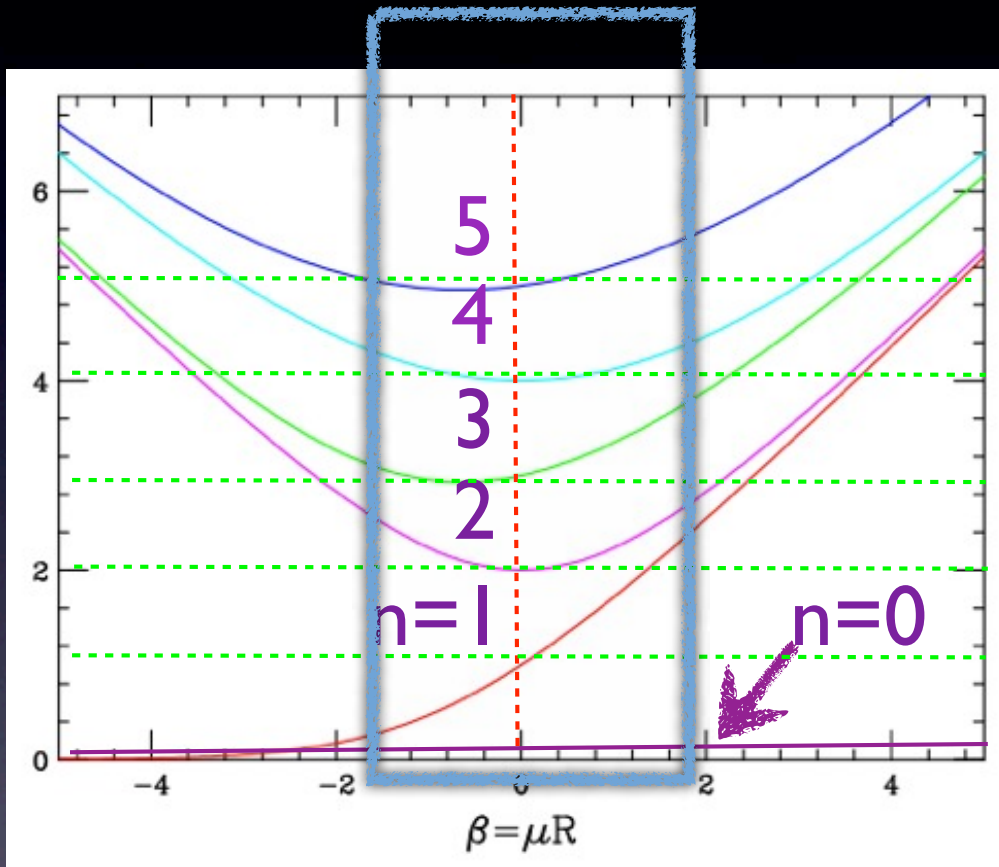
# Full KK spectra, DL

$$m_5(y) = \mu\theta(y)$$

ultralight  
mode



- $M_5=0$  corresponds to UED.
- Chiral zero mode remains massless ( $n=0$ ).
- KK mass spectrum of boson(fermion) is (not) equally spaced.
- $n=1$  mode can be degenerate with the zero mode (ultra light mode) when  $\mu$  is negative. If positive, it will be heavy and approaches to  $n=2$ .

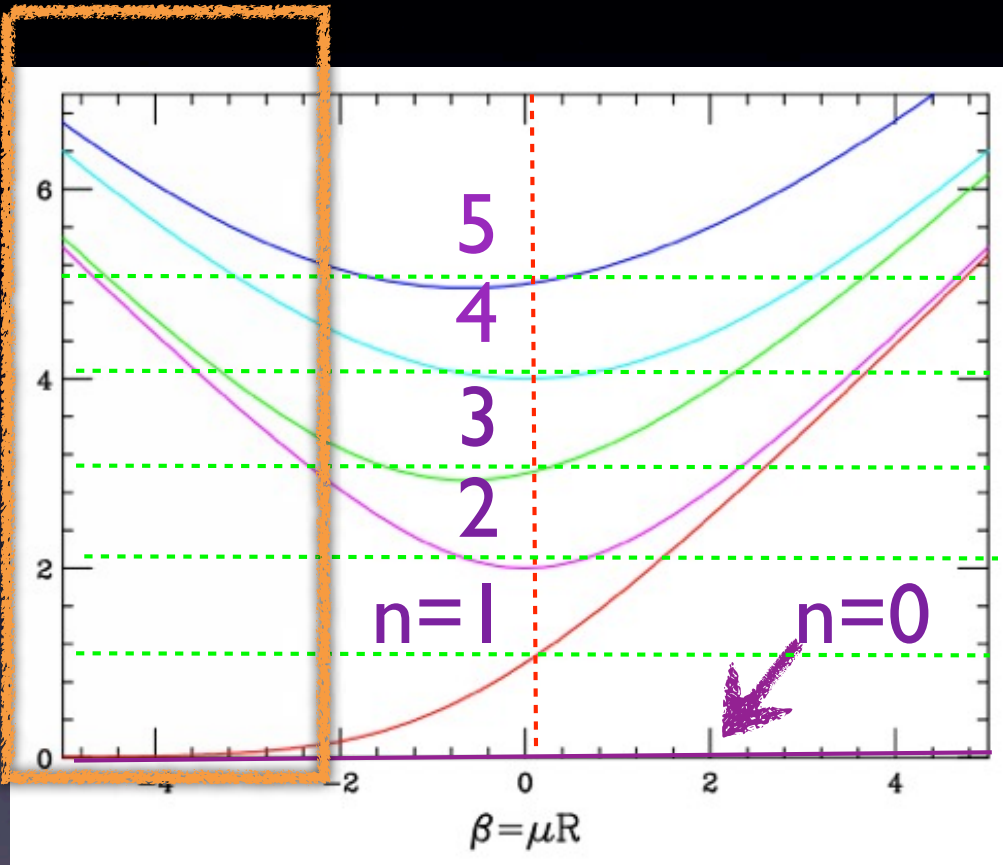


$$m_{\text{fermi}}^1 \sim m_{\text{gauge}}^1$$

- Similar to the conventional UED but details depend on the bulk masses
- If  $\mu > 0$ , LKP is the 1<sup>st</sup> KK boson
- KK states will be seen at the LHC with  $1/R \sim \text{TeV}$

Chen, Nojiri, SCP, Shu (2009, 2009)  
Kong, SCP, Rizzo (2010, 2010)

$$m_{\text{fermi}}^0 \lesssim m_{\text{fermi}}^1 \ll m_{\text{gauge}}^1 \ll m_{\text{fermi}}^2$$



- The “superlight modes” looks like the vectorlike 4<sup>th</sup> generation [Kong, SCP, Rizzo \(2010\)](#)
- Typically long lived. decay via KK gauge bosons which are much heavier than fermions.
- KK tower is not seen at low energy. Not quite looks like extra dimension..

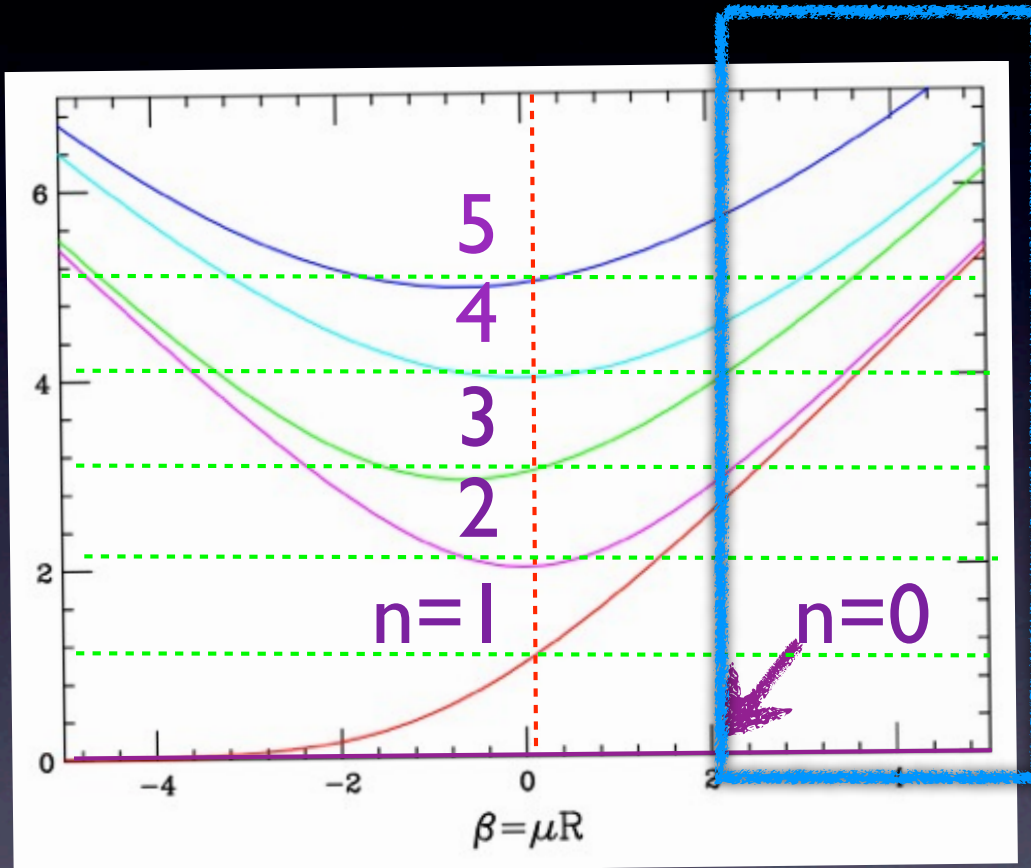
$$L^{-1} = 10^2 \text{TeV}, \mu = 7.3 L^{-1}$$

$$L^{-1} = 10^{11} \text{TeV}, \mu = 29 L^{-1}$$



$$m_1 = 1 \text{TeV}$$

$$m_{\text{fermi}}^0 \ll m_{\text{gauge}}^1 \ll m_{\text{fermi}}^1 \sim m_{\text{fermi}}^2$$



- Tower of KK-gauge bosons appears well before touching the first KK-fermion
- similar to the case where only bosons are in the bulk
- Collider signature?

Finally

# Flavor hierarchy in split-UED + Localized Higgs

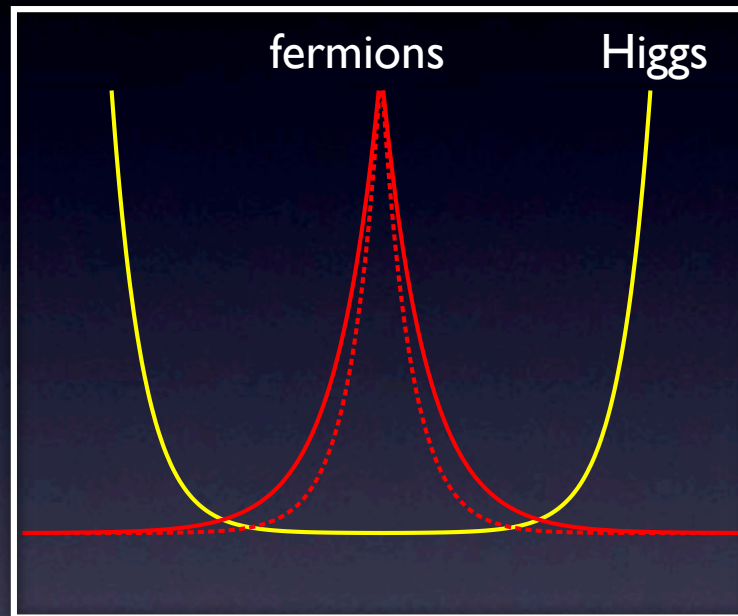
Csaki, Hubisz, Heinonen, SCP, Shu (arXiv:1007.0025)

- We might be more ambitious and try to address Yukawa hierarchy problem.
- One way to get the hierarchical overlap with the Higgs required to naturally generate the hierarchy of fermion masses is to **localize the Higgs in a KK-parity symmetric way** on the end points of the interval.

Our model Higgs [split-UED']

$$V_L(H) = V_{-L}(H) = \lambda(|H|^2 - v^2)^2$$

$$S = \int d^5x |D_M H|^2 - m_H^2 |H|^2 - \delta(y+L)V_{-L}(H) - \delta(y-L)V_L(H)$$



$$v(y) = A \cosh(m_H y)$$

- We found the lowest energy VEV profile is indeed localized toward end points: a perfect situation for generating hierarchy. So far so good...but...

# Bound from $K$ - $\bar{K}$ mixing

- **No RS-GIM** like mechanism works in UED so that flavor bound is severe as we can expect.

$$C_K^4 \approx \left[ \frac{L \cdot 500\text{GeV}}{1000\text{TeV}} \right]^2 \quad \text{induced by flavor changing KK gluon exchanges}$$

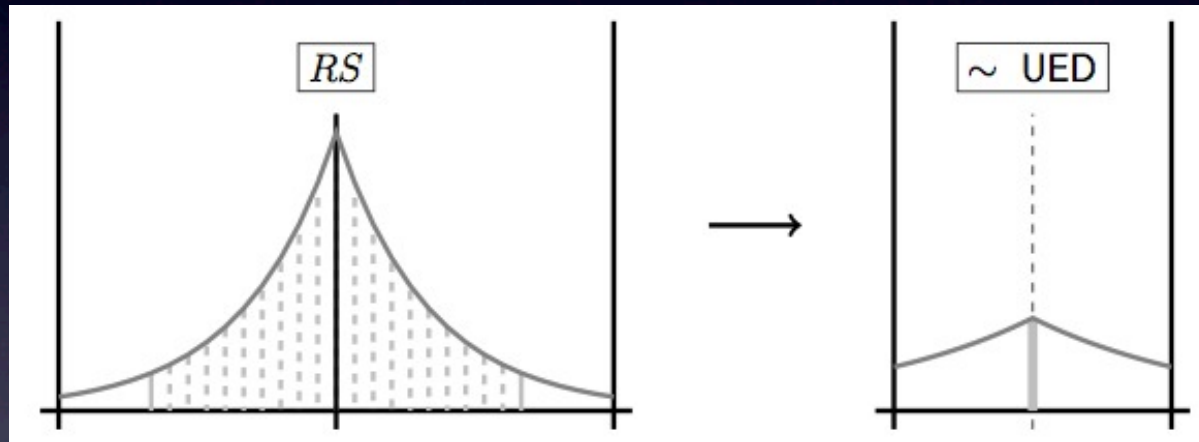
$$\begin{aligned} \text{Re}[C_K^4] &\leq (10^4\text{TeV})^{-2} \\ \text{Im}[C_K^4] &\leq (10^5\text{TeV})^{-2} \end{aligned} \quad \Rightarrow \quad L^{-1} \geq 500\text{TeV}$$

Disappointingly large KK scale is required to fit flavor bound if you are ambitious to explain flavor hierarchy problem in UED .

Q. How to make KK-scale low so that the model remains interesting for the LHC search?

# RS to UED

Interpret UED as an effective description of RS with two throats [a successful model of flavor]

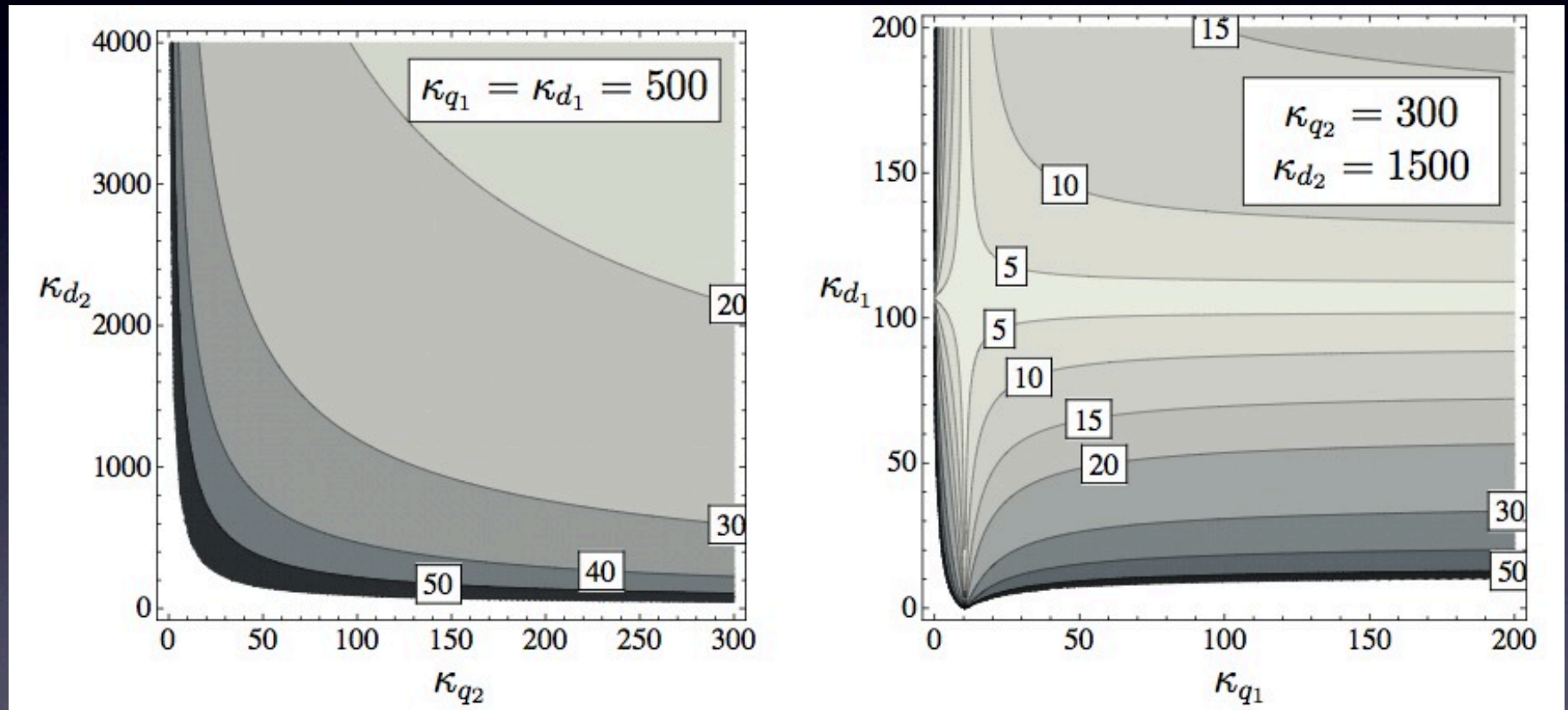


- It is possible to reduce a warped geometry to an approximately flat XD by integrating over a large slice of the warped XD. A new BLKO induced.

$$S_{\text{fermion}} = \int d^5x \left\{ \frac{i}{2} \bar{\Psi} \Gamma^\mu \overleftrightarrow{\partial}_\mu \Psi \right\} \kappa_f L \delta(y).$$

- The remaining warping is minimal and it is clear that this model will describe exactly the same physics as the complete warped XD, encapsulating RS-GIM mechanism.

# Allowed range



- Large BLKT allows rather low KK-scale ( $\sim 5$  TeV)

# Summary

- Split-UED allows 5D masses in a way of keeping KK-parity.
  - Phenomenology becomes richer.
  - Flavor hierarchy may be due to the non-flat profiles but stringently constrained because of lack of RS-GIM like mechanism.
  - As a way out we suggest to regard UED as an effective theory of two throats RS model and introduce BLKTs accordingly. There may be other ways e.g flavor symmetry to make the model MFV
- ✱ .. there may be more .. it is an open question .. we should talk more on this.

## Final exam)

- KK-parity requires flat geometry.  
Yes or No?
- KK-parity forbids 5D fermion mass.  
Yes or No?

Send your answer to  
[seongchan.park@ipmu.jp](mailto:seongchan.park@ipmu.jp)